Validity of the *V* parameter for photonic quasi-crystal fibers

Han Zhao,[1] Remo Proietti Zaccaria,[1,2,*] Prabhat Verma,[3] Junfeng Song,[1] and HongBo Sun[1]

[1]*LaSUN, State Key Laboratory on Integrated Optoelectronics, College of Electronic Science and Engineering, Jilin University, Changchun, China*

[2]*Italian Institute of Technology (IIT), NanoBioScience Laboratory, Genova, Italy*

[3]*Department of Applied Physics, Osaka University, Osaka, Japan*

**Corresponding author: remo.proietti@iit.it*

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We have investigated the validity of the *V* parameter to identify the single-mode operation regime for photonic quasi-crystal fibers. Our results show that the *V* parameter can be considered only for smoothly changing fibers. On the other hand, for fibers showing isolated high refractive regions around the core, only a mode area analysis can allow the estimation of the threshold for the single-mode operation regime. Finally, these findings can be generalized to any kind of optical fiber. © 2010 Optical Society of America

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The propagation of light inside optical fibers is commonly explained through either Bragg diffraction or the phenomenon of total internal reflection. In this Letter, we will focus on total-internal-reflection-like fibers, also known as index-guiding fibers (IGFs), where core propagation is achieved when the refractive index of the core is higher than that of the cladding. Further, we will discuss one particular family of IGFs having cross-sectional domains of quasi-crystal. One of the most important properties associated with a fiber is its capability of being either single-mode or multimode. In the case of single-mode fiber, for a given wavelength and polarization, only a particular mode can propagate inside the fiber. This avoids the problem of the destructive interference that occurs in a multimode fiber, which makes single-mode fibers the best choice for applications related to long-distance communications. Whether a particular IGF is single-mode or multimode for a given λ can be determined by looking at the extension of the second-order mode [1]. If the second-order mode is confined within the core of the fiber, then the fiber can propagate this mode and hence behaves as a multimode fiber. Thus a drastic increase in the second-order mode area can indicate a multiple-to-single mode transition. However, the increase for the second-order area is not always very steep. For this reason, the threshold value of λ for this transition is usually defined by calculating the so-called *V* parameter [2]. This quantity is closely related to the geometry of the fiber as well as to the wavelength of the incoming light. More specifically, for a standard optical IGF, we have $V(\lambda) = (2\pi a/\lambda)[n_{co}^2(\lambda) - n_{cl}^2(\lambda)]^{1/2}$, where λ is the wavelength of the incoming light, a and n_{co} are the radius and the refractive index of the core, respectively, and n_{cl} is the refractive index of the cladding. This well-established single–multiple-mode description has been extended to the regime of photonic crystal IGFs. This category of fibers presents a geometry that is completely different from the standard fibers. In fact, their profile follows a photonic crystal (PC) behavior, where holes drilled in a dielectric respect a pattern defined by a translational periodic modulation of the refractive index. This implies the necessity of redefining the *V* parameter for PC fibers with corresponding effective values, which then assumes the form [3]

$$V_{eff}(\lambda) = \frac{2\pi A}{\lambda}[n_{co,eff}^2(\lambda) - n_{cl,eff}^2(\lambda)]^{1/2}, \qquad (1)$$

where A is the lattice constant of the PC. In another study [4], the cutoff properties of PC fibers have also been explored within a slightly different formulation of the *V* parameter; however, here we focus our attention on the definition given in [3]. While PC fibers have received a great deal of effort [3–7], in particular to establish that the single-mode operation regime is achieved for $V_{eff} < \pi$, another kind of optical fiber, namely, photonic quasi-crystal fibers (PQFs), have not received the same kind of attention, even though they are known to play an important role in fiber optics applications [8]. This kind of IGF possesses a geometry that resembles that of its PC counterparts but with no translational periodicity. Only rotational and mirror symmetries can be identified in these structures. Here, we will discuss whether the definition of the *V* parameter, as in Eq. (1), can discern single-mode from multimode operations for *any* kind of PQF. As described in [3], the cutoff value $V_{eff} = \pi$ for a hexagonal IGF originates in identifying the pitch as the *natural length scale* of the structure. By extending these considerations to the geometries introduced in this Letter, we can see that π is confirmed as the V_{eff} cutoff. However, since the cross-sectional geometry of the fiber is quasi-crystal-like, one would expect a dependence of $n_{cl,eff}$ on the area of the cross section (namely, on the cladding of the fiber), resulting in a dependence of V_{eff} on the area of the cladding. This would preclude the possibility of using the *V* parameter to discern single- from multiple-mode fibers.

To understand this assumption, we have investigated three kinds of PQFs, the cross-sectional geometries of which are illustrated in Fig. 1. The numeri-

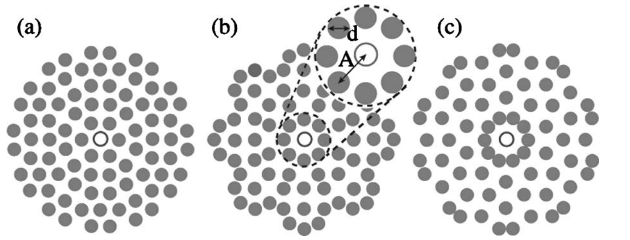

Fig. 1. Cross section of three kinds of PQFs: (a) sixfold, (b) eightfold, and (c) tenfold. Gray filled circles denote air holes drilled in silica fiber. The central hole (empty circle) is absent in the fibers. The quantities A and d represent the lattice constant and the hole diameter, respectively.

cal calculations used in this study use a 3D (vector) plane-wave expansion method in the framework of supercell description, which is considered with a convergence analysis. First, we discuss the sixfold quasi-crystal geometry, shown in Fig. 1(a), which is composed of square and triangular basic units [8]. The geometrical parameters used in the calculations are the lattice constant $A=2.4\ \mu$m and $d/A=0.7$, where d represents the diameter of the air holes. The material of the fiber is silica ($n=1.444$). Following Eq. (1), we have evaluated the V_{eff} parameter of this sixfold PQF, which is shown by the black squares and the black curve (best fitting) in Fig. 2(a). The x axis shows A/λ, and the horizontal dotted line is drawn at the point where this V_{eff} is π. The value of λ at the intersection of the V curve and the π line should define the cutoff wavelength for single–multiple-mode operation regimes. The other way to estimate this cutoff is to look for a sudden increase in the extension of the second-order mode by changing the excitation wavelength [1]. If this sudden change occurs at the same value of λ where the π line intersects the V curve, one can conclude that Eq. (1) is still valid and π could still represent the threshold for the single-mode operation regime. The simulation results for the extension of the second-order mode for a sixfold PQF is shown by the red circles and the red curve (best fitting) in Fig. 2(a). We can actually see how the second-order mode area, normalized to A^2, increases drastically at about $A/\lambda=2.5$ ($\lambda=0.96\ \mu$m) from 1 to at least 7.5. This is an undoubted proof that at wavelengths longer than about 1 μm the fiber starts shifting to the single-mode operation regime. Indeed, the three curves intersect at the same value of λ, confirming that Eq. (1) is valid for the sixfold PQF.

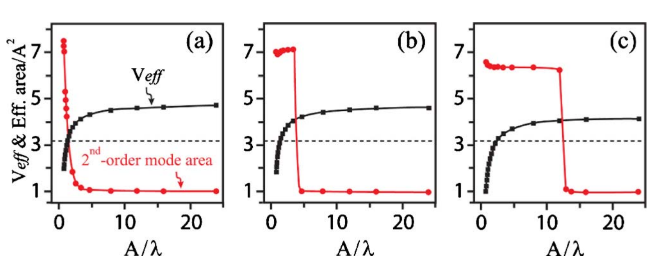

Fig. 2. (Color online) Effective V parameter (black squares and curves) and the second-order mode normalized area (red circles and curves) as a function of A/λ for (a) sixfold, (b) eightfold, and (c) tenfold quasi-crystal fibers. The dashed lines are drawn at π.

The first result for the sixfold PQF seems to support the validity of Eq. (1) for PQFs, going against our predictions. We then tried to verify the generalization of this result by investigating the eightfold and tenfold PQFs, cross-sectional geometries of which are illustrated in Figs. 1(b) and 1(c), respectively.

The eightfold quasi-crystal structure is formed by square and rhombus basic units. The geometrical parameters used in the calculation of this structure are $A=2.4\ \mu$m and $d/A=0.7$. Once again we consider the V_{eff} parameter and the second-order mode as a function of the normalized effective area. The results are shown in Fig. 2(b). By looking at the effective area, we find a single-mode cutoff at $A/\lambda=4.0$ ($\lambda=0.6\ \mu$m), which is quite different from $A/\lambda=1.5$ ($\lambda=1.6\ \mu$m) obtained from the intersection of the V curve with the π line. This shows a strong disagreement between the two methods of finding the cutoff wavelength. We then analyzed the tenfold PQF, shown in Fig. 1(c). The calculated results are shown in Fig. 2(c), where $A=2.4\ \mu$m and $d/A=0.6$. Similar to the previous case, in this case also we do not find a match between the cutoff for single-mode operation regime coming from V_{eff} and the second-order mode area, indicating that Eq. (1) is not valid even for the tenfold PQFs.

To understand this mismatch, we have looked at the modes of the fibers. For simplicity we will show only the results for the eightfold PQF; however, the conclusions are also valid for the tenfold structure. We have calculated the modes at various values of λ at short intervals to understand how the effective area of the mode depends on the wavelength. Figures 3(a) and 3(b) show the second-order modes for an eightfold PQF at two different values of λ, where a sudden jump in the mode area is observed. This jump corresponds to the results seen in Fig. 2(b), where, unlike for the sixfold PQF, the second-order mode shows a steplike behavior. The two wavelengths ($\lambda=0.50\ \mu$m and $\lambda=0.75\ \mu$m) correspond to two points across this step.

In Fig. 3(b), light is confined mainly in a ring formed by square basic units surrounding the core of

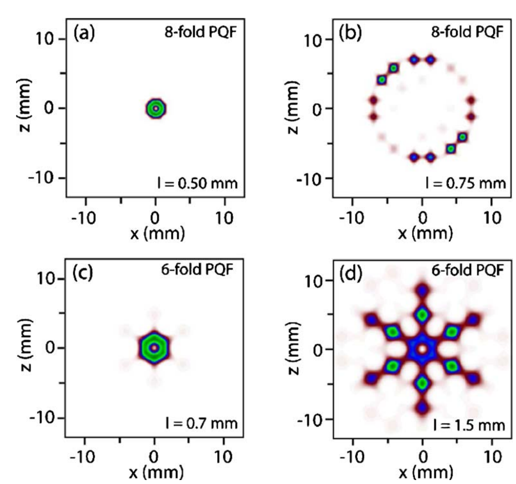

Fig. 3. (Color online) Simulation results for the second-order modes for (a), (b) eightfold PQF and (c), (d) sixfold PQF, at indicated wavelengths.

the fiber. Interestingly, no light exists in the rhombus unit cells. Both the fact that light prefers square units versus the rhombic units and the existence of a ring formed by square units makes the light exist either in the central core for small wavelengths [Fig. 3(a)], or in the external ring [Fig. 3(b)], with a sudden jump when a specific threshold wavelength is reached. Since the second-order mode undergoes a sudden jump at a particular value of λ, it can be used to quantify the cutoff value of wavelength for single-to-multiple-mode transition for the eightfold and the tenfold PQFs. This behavior is very different from the standard fibers, the PC fibers, or even the hexagonal PQF, where light leaves the core without any abrupt change. For example, we calculated the second-order modes for the sixfold PQF at $\lambda = 0.7\ \mu m$ and $\lambda = 1.5\ \mu m$. The results are shown in Figs. 3(c) and 3(d), respectively. As can be clearly seen, the mode at $\lambda = 1.5\ \mu m$ is much more extended than the mode at $\lambda = 0.7\ \mu m$. More important, the mode is not localized in only the ring surrounding the fiber core, confirming that there are no sudden jumps in this case.

Because light prefers regions of higher refractive index, and because the effective refractive index for square units is higher than that for the rhombus units, little or no signal is found in the rhombus units. Moreover, following a waveguide description, the increase in wavelength will reduce the V parameter of the fiber, which, at a certain point, will forbid the existence of any mode besides the fundamental. The second-order mode will then find its own way outside the central core by concentrating on the first squarelike external ring. It is also interesting to notice that for the eightfold and tenfold structures the fiber starts working in single-mode at smaller wavelengths than for the sixfold. This behavior, which is not strictly dependent on the fact that here we are dealing with quasi-crystal structures [9], is related to the decrease of the effective core index when the folding is increased.

To confirm this description, we fabricated a traditional waveguide having two rings similar to a coaxial fiber. The structure is shown in the insets of Fig. 4. This fiber, with its inner and outer rings having same respective radii and effective refractive indices as the core and the outer ring made of the square unit cells in the eightfold PQF, should imitate the optical properties of the eightfold PQF. Simulation results for this coaxial fiber are presented in Fig. 4, which shows a behavior similar to the eightfold PQF. In fact, the second-order mode is confined in the center of the fiber for $\lambda = 0.50\ \mu m$, whereas hardly any signal can be found in the core for $\lambda = 0.75\ \mu m$, with most of the light confined in the ring. This helps us to understand the physical process in Figs. 3(a) and 3(b).

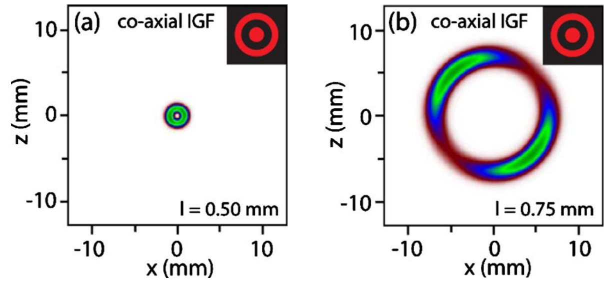

Fig. 4. (Color online) Simulation results for the second-order mode of a traditional coaxiallike fiber at indicated wavelengths.

In conclusion, we have investigated the validity of the V parameter defined in [3] as a useful tool for identifying the single-mode operation regime in case of quasi-crystal optical fibers. We have demonstrated that it can be utilized only for smoothly changing fibers that do not have isolated high-refractive regions around the core. On the other hand, if such high-refractive regions do exist in the fiber structure, the V parameter approach looses its validity, and only a mode area analysis can identify the threshold value of λ for single-mode behavior of the fiber.

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